

Worksheet 7

MATH 33A

- (a) Find the eigenvectors and eigenvalues of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.
(b) Compute $\det(A)$, $\text{tr}(A)$. What do you notice about how these are related to the eigenvalues of A ?
- Diagonalize the above matrix. Compute A^{100} . Find $\det(A^{100})$, $\text{tr}(A^{100})$.
- Find the eigenvalues of the 100×100 matrix

$$\begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix}.$$

- Show $\text{tr}(AB) = \text{tr}(BA)$ for any two $n \times n$ matrices A, B .
- Let $A = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$. Is it possible to find a basis for \mathbb{R}^2 consisting of eigenvectors of A ? If so, compute such an eigenbasis.