Worksheet 6

MATH 33A

1. Use the Gram-Schmidt process to find the orthonormal basis corresponding to the basis $\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \end{bmatrix} \right\}$ of $\mathbb{R}^2$. Use this to find the QR decomposition of $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.
2. Show that the composition of two isomorphisms $T, T' : \mathbb{R}^n \to \mathbb{R}^n$ is an isomorphism.
3. Use the linear transformation of $(x, y, z) \to (ax, by, cz)$ to show that the volume of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ is $\frac{4}{3}\pi abc$.
4. Find all the possible eigenvalues of a 3×3 matrix A satisfying $A^3 = 1$.