Worksheet 4

MATH 33A

1. Let W be the subspace in $\mathbb{R}^3$ spanned by the plane $x + 2y + z = 0$. Find $\dim W$ and find a basis for W. How many elements are in the basis?

2. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation given by the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 3 & 0 \end{bmatrix}$. Find $\operatorname{im}(T), \ker(T), \operatorname{rank}(A)$. Are the vectors that form the columns of A linearly independent? Do they span $\mathbb{R}^3$?

3. Let $X = \{e_1, ..., e_n\}$ be a basis for $\mathbb{R}^n$ and $X = \{w_1, ..., w_n\}$ be another basis. Let $T : \mathbb{R}^n \to \mathbb{R}^n$ be a linear transformation. Show that $T_B = PT_A P^{-1}$, where P is the matrix whose columns are $w_1, ..., w_n$.

4. Find an orthonormal basis for the subspace of $\mathbb{R}^4$ spanned by $\operatorname{im}(A)$, where

$$A = \begin{bmatrix} 0 & 1 & 1 & 2 \\ -2 & 3 & 0 & 1 \\ 1 & 1 & -1 & 1 \\ 3 & 2 & -2 & 3 \end{bmatrix}.$$